SCALING IN MODELLING GLOBAL POPULATION GROWTH

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ABSTRACT

Global population growth is described over practically the whole of human history by assuming self-similarity as the dynamic principle of development. Nonlinear growth is proportional to the square of the number of people due to a collective interaction of an informational nature. Estimates of the beginning of human development 4 – 5 million years ago and of the total number of people who have ever lived since then ≈ 100 billion are made. Large scale cycles, defined by history and anthropology, are shown to follow an exponential pattern of growth, culminating in the demographic transition. A veritable revolution, when global population growth is to stabilize at 10 – 12 billion in the foreseeable future.

Key words: global problems, population dynamics, scaling, asymptotic methods, phase transitions, demographic transition, social evolution,

1. Introduction

The subject of demography is the growth and development of populations explained in terms of their economic and social history, and of all global problems world population growth is the most significant. But in modern theories of complex dynamic systems global demography has been conspicuously absent. It was never seen as an object of such studies, as the population of the world was taken to be a mere arithmetical sum of all populations of countries and regions having no inherent dynamic meaning. But right from its origin mankind appeared as a system of interdependent and interacting entities. It acts as an evolving and growing dynamic system, when the growth of the number of people expresses the sum outcome of all economic, social and cultural activities that comprise global history seen at large. By applying the concepts of nonlinear dynamics it is possible to work out a mathematical model for a phenomenological statistical description of global population growth and project its development into the future.

Global modeling originated when an attempt to take into account the main factors responsible for the growth and development of the global system was made. The approach was pioneered by Jay Forrester and is mainly known by the first reports to the Club of Rome by Meadows (1972) and Mesarovic (1977). In these studies complex computer-based models, using inputs from extensive databases were developed. These models did provide insight into the growth of the global system and the studies drew attention to global problems opening up a new field of research. In these models there was no explicit connection between development and population growth. At present, the results of these initial models as studies in complexity have been extensively criticized and to address this 'curse of complexity' Herbert Simon has noted that:

> Forty years of experience in modeling complex systems on computers, which every year have grown larger and faster, have taught us that brute force does not carry us along a royal road to understanding such systems… modeling, then calls for some basic principles to manage this complexity.

In the following model a holistic and interdisciplinary approach to the development of the human population is explored rather than the reductionist treatment practiced in demography and most other social studies.

2. The demographic problem

The aim is to present the demographic problem in mathematical terms. The treatment will be as simple as possible, without sacrificing the meaning and content of the problem. The point is to focus on the physics of demographic growth, and then find the appropriate mathematical concepts to express these processes in terms of a phenomenological theory of many-particle system.

In this model humankind is treated as an entity, not breaking it up into countries and regions, an attitude which has been emphasized by those historians –Jaspers (1949), Konrad (1967), Wallerstein, Diakonoff (2000), who have consistently noted that fundamentally history can only be understood on a global scale. Braudel (1980) explicitly stated by that 'there are no substantial truths concerning humans, except on a global level'.

The population of the world at any given moment of time T will be characterized by a function $N(T)$. N is an additive variable and the only one taken into account to describe the global population. All other variables – the distributions of population in countries, towns or villages, by age and sex, by income and resources etc., in the first approximation are not taken into account. Then *N* is the senior variable, subordinating all others, the parameter of order discussed by Haken (1983).

For the demographic problem this corresponds to the principle of the demographic imperative, introduced to emphasize this concept Kapitza (2006). By singling out the principle senior variable *N* all other variables are averaged, when in the 'short' equation only two variables *T* and *N* are finally left:

$$\frac{\partial N_i}{\partial T_i} = \overline{F(N_i, T_i, X_i, Y_i, K_i, \tau_i, \nabla^2 N_i, ...)} \rightarrow \frac{dN}{dT} = f(N, K, \tau) \qquad (1)$$

with *K* and τ are scaling factors for population and time. For the demographic problem asymptotics means dealing with changes in time longer than that of a generation. Then it may be assumed that asymptotically birth and death rates, the duration of a lifetime will not explicitly enter into the formulas. This is the result of scaling in justifying the asymptotic approach to history, recognized by Braudel (1972), when he discussed the contradiction of *structure* and *conjunction* as long-term and short-term realities. In fact, all figures in the paper are drawn at different scales to show the perception of change.

In dealing with systems, having many degrees of freedom, where many factors affect growth it can be assumed that this multitude of processes could be treated statistically and that the nature of these statistics does not change. Then systemic growth will be self-similar, and, as a consequence of dynamic self-similarity, scale when the relative growth rate is constant:

$$\lim_{\Delta N, \Delta T \to 0} \frac{\Delta N}{(N - N_a)} \frac{(T - T_b)}{\Delta T} = \frac{d \ln| N - N_a |}{d \ln| T - T_b |} = \alpha \qquad (2)$$

and N_a and T_b are the initial values for population and time. Then self-similar growth

$$N = C(T - T_1)^{\alpha} \qquad (3)$$

is described by a power law, *C* and α are constants, and time is reckoned from T_1.

3. The case of quadratic growth

The appropriate description of growth of the global population system was proposed by a number of authors. One of the first was Forster (1960), then Horner (1965), and the Russian astrophysicist Shklovski (1987) suggested as an empirical formula:

$$N = \frac{C}{T_1 - T} = \frac{200 \cdot 10^9}{2025 - T} \qquad (4)$$

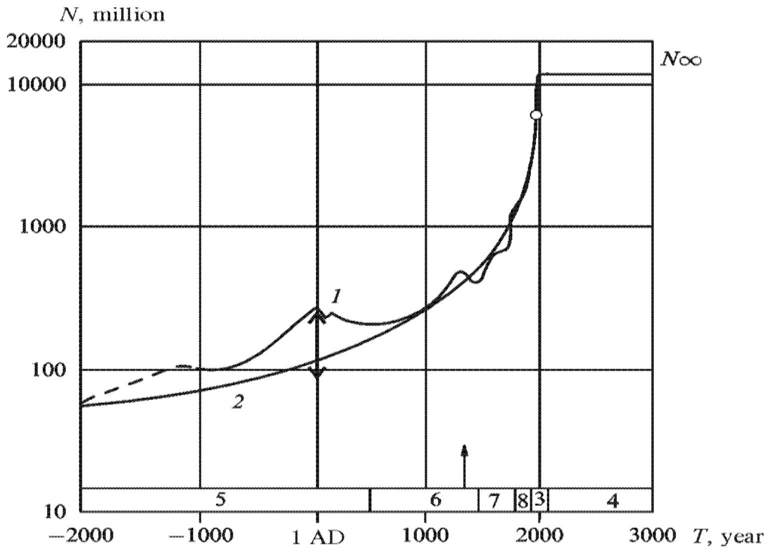

Figure 1. World population from 2000 BC to 3000 AD. Limit $N_{\infty} = 12$ billions
1 – data Biraben (1979), 2 – blow-up growth, 3 – demographic transition, 4 – stabilized population, 5 – Ancient world, 6 – Middle Ages, 7 – Modernity, 8 – Recent history, ↑ – the Plague, ◦ – 2000.
As the demographic transition is approached, the time of history and development is compressed.

describing the growth over a very extensive time. Self-similar growth is limited by the blow-up singularity in 2025 indicating that we are now approaching a crisis in the growth of human numbers. At the other extreme 20 billion years ago it indicates that then there would have been some ten cosmologists around, discussing the Big Bang! These considerations added to the arguments of the demographers in not taking this hyperbolic expression as meaningful.

Independently this expression was obtained by the author, who right from the beginning recognized it as an asymptotic equation describing self-similar growth limited in time by the human life span. It was a decisive step as then the human life time was brought into the model and the limits of scaling and intermediate asymptotics could be established, as discussed by Bahrenblatt (1996). From equ. (4) it follows that the growth rate has a singularity at T_1:

$$\frac{dN}{dT} = \frac{C}{(T_1 - T)^2} \tag{5}$$

For the main epoch **B** before blow-up $\alpha = -1$. This corresponds to a quadratic growth rate, proportional to N^2, described by the asymptotic autonomous equation of growth:

$$\frac{dN}{dT} = \frac{N^2}{C} \tag{6}$$

expressing the growth rate as a function of the state of the system. This nonlinear equation indicates the presence of a collective interaction, acting in the global population system. The quadratic interaction is non-additive and it cannot be directly applied to any part of the system or country. As all people belong to the same species, they interact, move around and inhabit the whole Earth, all places fit to live in. In this case migration does not explicitly contribute to the global population and in a finite world in the first approximation this implies that the model is non-local. As numbers are averaged in time it also implies a memory in the representation of the system.

Instructive analogies for global population studies are provided by the kinetic theory of gases. In an ideal gas all states are self-similar: hence the gas laws scale. In thermodynamics of gases and kinetics of population growth temperature and time are the parameters, determining the changes in these systems, one closed and conservative, the other open and evolving. In a non-ideal gas model the Van der Waals interaction sums up all binary interactions between molecules and is proportional to the square of the density, the square of the number of particles in a given volume and sets the limits of self-similar states by phase transitions. In a similar way the growth of human numbers is due to collective interaction proportional to the square of the total population on a finite Earth. This is seen to be due to an exchange, mainly by language, of generalized information throughout the whole development of mankind, and is peculiar for the mind and consciousness of *Homo*.

The self-similar pattern of growth is not unbounded and can happen only within limits set by the discrete nature of population – it cannot be less than one person and by the shortest unit of time – by the scale of the effective length of a generation. In other words the limits of self-similar growth are defined by singularities.

The first was the appearance of *Homo*. This was a critical event in human development as it seemed to have happened as an evolutionary discontinuity, for no one has ever found the proverbial 'missing link between man and ape'. But the recent discovery by Pollard K. et al, (2006) of a gene responsible for controlling the growth of the human brain may explain how a critical mutation appeared 5 to 7 million years ago and suddenly changed the limits of the brain to grow. The outcome of this mutation during the initial epoch **A** of anthropogenesis some 3 – 4 million years ago was the emergence of hominids with a brain and mind that provided a radically new capacity to develop and grow faster than all other creatures. This decisive event in human evolution led to the appearance 1.6 million years BP of some 100 000 hominids. Since then quadratic growth gradually took over as the result of the emergence of behavioral patterns of passing information to the next generation and spreading it worldwide. The next comparable critical event became the singularity of the demographic revolution.

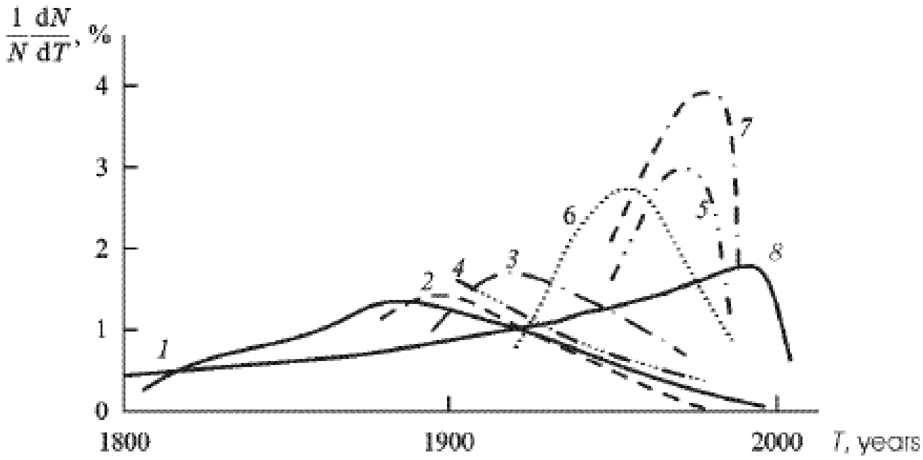

Figure 2. Relative growth rate during the demographic transition for:
1 – Sweden, 2 – Germany, 3 – USSR (Russia), 4 – USA, 5 – Mauritius, 6 – Sri Lanka, 7 – Costa Rica, 8 – global Model. The data are smoothed, so as to show the general trends. Compare with Fig. 3

4. The demographic transition

The demographic transition is a crucial event in the growth of human populations. It was discovered by the French demographer A. Landry first for France, where, after incessant growth of human numbers a marked decrease in growth was observed. Following the tradition in demography the demographic transition was interpreted in terms of local social and economic conditions, for example discussed by Chesnais (1992).

The change in the rate of procreation is rapid and universal and has a major effect on all processes, which go on in society as its population stabilizes. This led Landry to consider the demographic transition as a demographic revolution. Moreover it is really a global phenomenon, when all countries of the world are simultaneously engaged in the transition, in spite of the great difference in local economic and social conditions. In fact, the developing world is passing through the transition twice as fast as did the developed world only 50 years earlier, as a powerful global factor is synchronizing and narrowing the transition. This makes the demographic revolution the greatest crisis in the history of humankind, when from unlimited procreation the world rapidly changes into a population growing no more. This change taking less than a 100 years can be interpreted as a phase transition in an evolving system.

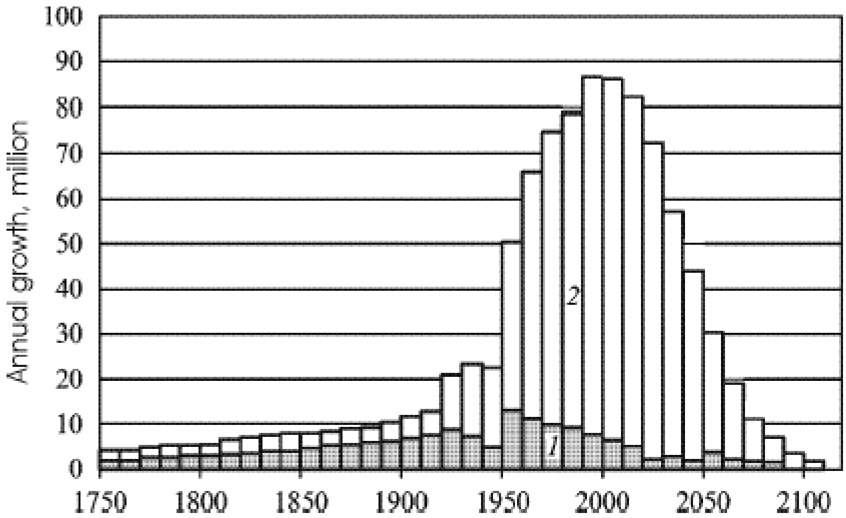

Figure 3. The global demographic transition 1750 – 2100
Annual growth averaged over a decade. 1 – developed countries, 2 – developing countries

The main point is to establish the limits of scaling in equation (4) and exclude the blow-up singularity and the singularity in the distant past. These conditions are:

$$\left(\frac{dN}{dT}\right)_{\min\left|\frac{N\to 1}{T\to T_0}\right.} \geq \frac{1}{\tau_0}, \qquad \frac{1}{\tau_1} \geq \left(\frac{1}{N}\frac{dN}{dT}\right)_{\max|T\to T_1} \tag{7}$$

and indicate, that the limits in equ.(2) cannot be reached, assuming that the minimal growth rate of human numbers is limited by integers to one person appearing in τ_0, and at T_1 – by the maximum growth during the intrinsic time τ_1 and setting the limits of scaling. The first limit is valid for the initial stages of development, and the other – for the present. Demographic and anthropological data shows that $\tau_1 = \tau = \tau_0$ simplifying all calculations and indicating a common origin for the value of τ.

The equations for the growth rates are obtained by adding a term $1/\tau$ to (6). To cut off the hyperbolic singularity during the demographic transition a τ^2 term should be added in the denominator of (5) , where τ is the half-width of the transition:

$$\frac{dN}{dT} = \frac{N^2}{C} + \frac{1}{\tau} \tag{8a}$$

$$\frac{dN}{dT} = \frac{C}{(T_1 - T)^2} \tag{8b}$$

$$\frac{dN}{dT} = \frac{C}{(T_1 - T)^2 + \tau^2} \tag{8c}$$

When these equations are written in a dimensionless form, it will become obvious that the terms added are of a similar nature. Having brought τ into the equations of growth,

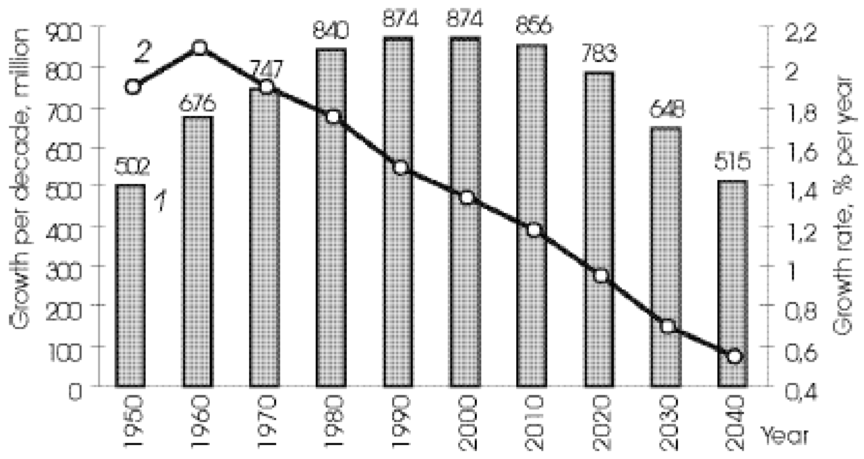

Figure 4. The global demographic transition. UN data (1995)
1 – growth, averaged in a decade, (left scale). 2 – relative growth rate in % per year (right scale).

the human life time enters as the microscopic parameter of the theory. It determines the minimal limit of the growth rate in the past and also regularizes divergent growth during the population explosion, extending it to into the future, beyond the critical date at T_1. When a finite width of the global population transition is brought in, the hyperbolic run-away critical time 2025 in (3) shifts to year 2000.

The constants appearing in these equations are determined by integrating (8c):

$$N = \frac{C}{\tau}\cot^{-1}\left(\frac{T_1 - T}{\tau}\right) \tag{9}$$

and then fitting this equation to population data. The best fit is the Model II with:

$$C = (172 \pm 5)\cdot 10^9,\ \ T_1 = 2000 \pm 2,\quad \tau = 45 \pm 2,\ \ \text{and}\ \ K = \sqrt{\frac{C}{\tau}} = 62000 \pm 1000. \tag{10}$$

If τ critically depends on the global demographic transition, calculations show, that T_0 and C do not significantly depend on τ, with a common value of τ leading to a plausible estimate for the beginning of anthropogenesis $T_0 = 4.4$ million years ago.

The relative growth rate:

$$\frac{1}{N}\frac{dN}{dT} = \frac{\tau}{\left[(T_1 - T)^2 + \tau^2\right]\cot^{-1}\dfrac{T_1 - T}{\tau}}, \tag{11}$$

reaches its maximum relative growth rate:

$$\left(\frac{1}{N}\frac{dN}{dT}\right)_{\max} = \frac{72.5}{\tau}\% \quad \text{per year} \tag{12}$$

at $T_m = T_1 - 0.43\tau$, or 1.6% in 1986, not taking into account a short time of postwar growth from 1950 to 1970:

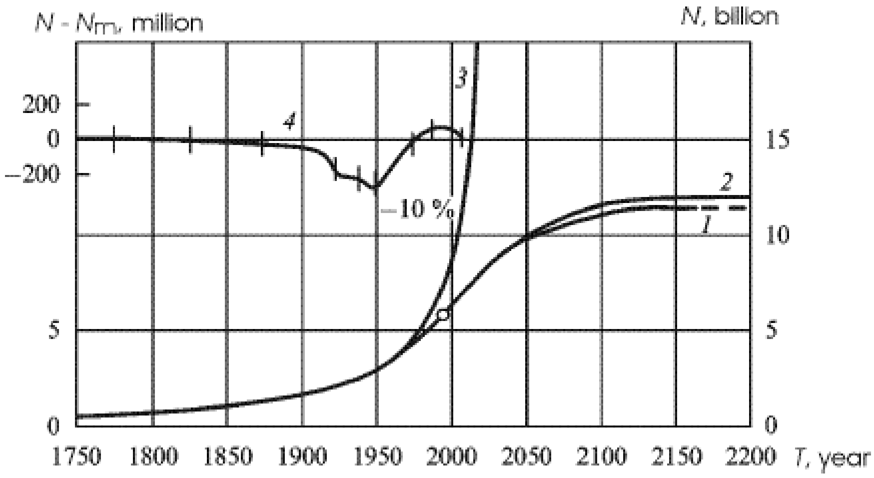

Figure 5. Population of the world from 1750 to 2200. ∘ – 1995.
1 –projections by IIASA and UN Population division, 2 – Model, 3 – blow up, 4 – difference between Model and global population, enlarged five times, showing losses due to World Wars I and II.

$$\left(\frac{1}{N}\frac{dN}{dT}\right)_1 = \frac{2}{\pi\tau} = \frac{63.6}{\tau}\% \quad \text{per year,} \tag{13}$$

that is less than (12) and equal to 1.4% in 2000. In the rapidly changing circumstances of the global transition the maximum for the relative growth rate, expressed in per cent per year precedes the high point for the absolute growth rate by 0.43τ = 20 years. Then the maximum growth rate in T_1 = 2000 becomes:

$$K^2/\tau = 86 \text{ million per year,} \tag{14}$$

or 240 000 people in every 24 hours. During the demographic transition from $T_1 - \tau = 1955$ to $T_1 + \tau = 2045$ the global population on the average will grow as:

$$\frac{\Delta N}{\Delta T} = \frac{\pi K^2}{4\tau} = 67 \text{ million per year} \tag{15}$$

The first cultural period of epoch **B** – the Olduvai – lasted a million years leaving only half of that time up to the present. The Middle ages lasted 1000 years and ended 500 years BP. This proportion for time elapsed and the exponential compression of time in history is valid all through the population explosion up to the demographic revolution. Any part of the global system isolated for a sufficiently long time will be retarded in its development. It is at this scale of time and growth we should ascertain the significance of the changes through which humankind is now passing as the outcome of the explosive quadratic collective mode of growth. Data for global population over the ages is summed up by Cohen (1997) and UN (2005) and, with the calculated numbers from 1.6 million years BP to year 2500, are presented in Table 1.

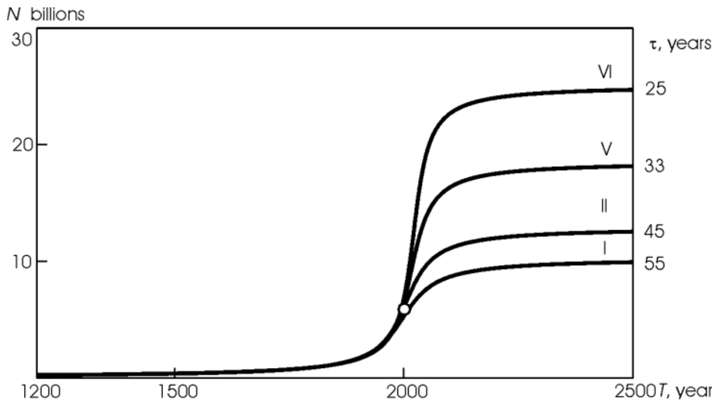

Figure 6. Population growth models for different values of the time constant τ

Table 1. World population N and result of modeling N_m (in millions for Model II)

Year	N	N_m		Year	N	N_m
$-4.4\cdot10^6$	(0)	0		1960	3039	3245
$-1.6\cdot10^6$	0.1	0.1		1965	3345	3497
– 35 000	1 – 5	2		1970	3707	3778
– 15 000	3 – 10	8		1975	4086	4089
– 7 000	10 – 15	16		1980	4454	4430
– 2 000	47	42		1985	4851	4801
0	100 – 230	86		1990	5277	5198
1000	275 – 345	173		1995	5682	5613
1500	440 – 540	345		2000	6073	6038
1650	465 – 550	492		2005	6453	6463
1750	735 – 805	685		2010	6832	6878
1800	835 – 907	851		2025	7896	7987
1850	1090 –1110	1120		2050	9298	9259
1900	1608 –1710	1625		2075	9879	9999
1920	1811	1970		2100	10400	10451
1930	2020	2196		2125	10700	10745
1940	2295	2474		2150	10800	10956
1950	2556	2817		2200	11000	11225
1955	2780	3019		2500	—	12000

The correspondence between recent data and calculations can indeed be impressive: For 2025: $N = 7965$, $N_M = 7987$ and for 2050: $N = 9298$, $N_M = 9259$ millions. Most current projections by demographers indicate a stabilization of the global population in the foreseeable future at 9 to 11 billion and substantiate modelling.

In epoch **C** after the global demographic transition a marked prevalence of the older generations is to be expected. By that time the most critical factor will be not the longer life expectancy, but the total fertility rate – the number of children per woman.

5. Dimensionless variables for time and population

Most results are seen best, if they are presented by dimensionless variables for time and population:

$$t = \frac{T - T_1}{\tau} \qquad (16)$$

$$n = \frac{N}{K} \qquad (17)$$

where time is measured in units of τ and N – in units of K. See equ. (10). The constant K is the large parameter of the global population system, which enters into all equations and results of the theory. When these new variables are used the differential equations of growth and their solutions are:

$$\frac{dn}{dt} = \frac{n^2 + 1}{K}, \qquad n = -\cot\frac{t}{K} \qquad (18a)$$

$$\frac{dn}{dt} = \frac{n^2}{K}, \qquad n = -Kt \qquad (18b)$$

$$\frac{dn}{dt} = \frac{K}{t^2 + 1}, \qquad n = -K\cot^{-1} t \quad \text{or} \qquad (18c)$$

$$\frac{dt}{dn} = \frac{t^2 + 1}{K}, \qquad t = -\cot\frac{n}{K}, \qquad (18d)$$

which become compact and the conjugate symmetry of time and population is seen, as well as how both of the hyperbolic singularities are dealt with. If equations **a** and **d** are compared near the singularity of the demographic transition at $t = 0$, population becomes, in a sense, the independent variable. Physical time certainly is invariant, but here systemic time appears as a variable, reciprocally connected with *n*. In other words, the moment of the demographic transition is determined by population growth and is not explicitly causally dependent on time. In a nonlinear theory systemic time can become a function of development and in social studies this has been long known. For example, anthopologists have traditionally projected the Stone Age on a logarithmic scale, for otherwise it is not possible to accommodate the Neolithic 10 000 years BP and the Lower Paleolithic, a million years ago on a linear scale. Although for the initial stages of anthropogenesis a linear scale for has been used by Jones et al. (1994)

Table 2. Growth and development of humankind on a logarithmic scale

Epoch	Period θ	Date year	Number of people	Cultural period	ΔT years	Events in history, culture, and technology
C		2200	11×10^9	Stabilizing global Population		Global population limit 12×10^9 Changing age distribution
		2050	9×10^9		125	Globalization
		T_1 2000	6×10^9	Global demographic Revolution	45	Urbanization, Internet
B	11	1955	3×10^9		45	Biotechnology Computers
	10	1840	1×10^9	Recent	125	World Wars Electric power
	9	1500		Modern	340	Industrial revolution Printing
	8	500 AD		Middle ages	1000	Geographic discoveries Fall of Rome
	7	2000 BC	10^8	Ancient world	2500	Christ, Muhammad Greek civilisation,Axial time India, Buddha,China,Confucius Mesopotamia, Egypt
	6	9000		Neolithic	7,000	Writing, Cities Bronze and iron metallurgy Domestication and agriculture
	5	29,000	10^7	Mesolithic	20,000	Microliths
	4	80,000		Moustier	51,000	America populated Shamanism *Homo sapiens*
	3	0.22 Ma	10^6	Acheulean	1.4×10^5	Language Speech, Fire mastered
	2	0.6 Ma		Chelles	3.8×10^5	Europe and Asia populated Hand axes
	1	1.6 Ma	10^5	Olduvai	1×10^6	Choppers *Homo habilis*
A	0	T_0 4 – 5Ma 5 – 7Ma	(1)	Anthropogene	3×10^6	Hominida separate from Hominoids Critical mutation affecting the Growth of the brain

In this case socially relevant time is the logarithm of Newtonian time. In the study of the temporal framework of history this has a very pronounced effect when comparing social processes happening at different stages of growth in the past, which become in socially relevant terms much closer. In a way it is similar to the transformation of time introduced in the theory of relativity, when describing physical events in different frames of reference. This is discussed by Prigogine (1980) in terms of *duration* and the direction of time – *the arrow of time* – for irreversible processes in evolving systems far from equilibrium. For the global population system the quadratic growth rate in epoch **B** this is expressed as in (5) by equating growth to development:

$$\frac{dN}{dt} = \frac{N^2}{K^2} , \tag{19}$$

This basic non-linear equation for the collective interaction describes growth covering five orders of magnitude. It indicates how growth is determined by development, where the N^2 term is a measure of the network complexity of the global population, or as a binary interaction similar to the Van der Waals interaction in a theory of non-ideal gases and many-particle systems in physics, with K as the form factor.

6. The limit of population and the number of people who ever lived

The solutions describing growth (7) leads to the global population limit expected to be asymptotically reached in the foreseeable future with an upper limit of

$$N_\infty = \pi K^2 = 12 \text{ billion} \tag{20}$$

and the beginning of growth at $t_0 = -\pi / 2 \cdot K$, or for an estimate of T_0 :

$$T_0 = T_1 - \frac{\pi}{2} K\tau = T_1 - \frac{\pi}{2}\sqrt{C\tau} = T_1 - \tau\sqrt{\pi N_1 / 2} = -4.4 \text{ millions years ago} \tag{21}$$

where the effective age of the humanity is expressed by $N_1 = 6 \cdot 10^9$ at $T_1 = 2000$ and τ. By integrating growth from T_0 to T_1 the number of people, who ever lived, is:

$$P_{0,1} = K\int_{t_0}^{t_{1/2}} \cot\frac{t}{K}dt + K\int_{t_{1/2}}^{0} K\cot^{-1} dt = \frac{1}{2}K^2 \ln K + \frac{1}{2}K^2 \ln(1+K) \cong K^2 \ln K \tag{22}$$

For the model $P_{0,1} = 2.25K^2 \ln K = 96 \approx 100$ billion people. The multiplier 2.25 = 45:20 appear because in integrating the effective length of life is $\tau = 45$ years, although the quoted numbers were obtained for an effective life span of 20 years. This estimate is in good agreement with estimates 80 to 150 billion by Weiss (1984). The latest estimate for $P_{0,1} = 106$ billion was made by Haub (2005). As it is often the case in demography, the accuracy implied is greater than can be expected. In the initial epoch **A** of anthropogenesis the number of early hominids was:

$$P_A = 2.25K\int_0^K \tan\frac{t'}{K}dt' = 2.25K^2 \ln\cos 1 \cong 5 \text{ billions} \tag{23}$$

These estimates do not depend much on the model, and are of general interest for anthropology and human population genetics.

For presenting global population growth all asymptotic comparisons should be made in a log$T$ – log$N$ space. (See Eqn.2). This plot is matched to hyperbolic growth and scaling, as the plot T – log$N$ is for describing exponential growth. (See Fig.1). When point zero of the time scale at $T_1 = 2000$, is approached the singularity of hyperbolic growth is removed by eliminating the interval around this peculiar moment.

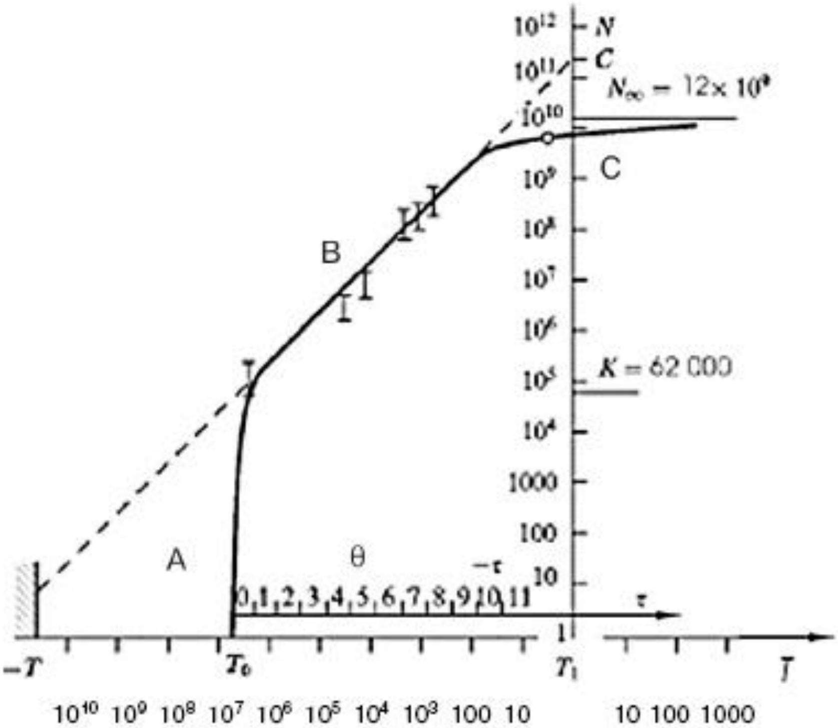

Figure 6. Global population growth from the origin of mankind and into the foreseeable future, described by the nonlinear model. $\theta = \ln t'$, ––– (4), ° – 1995

Growth dynamics may be described in terms of complex variables, which could help in justifying the model. But it would hardly help in interpreting the physics and social issues of growth and be beyond the means of those less equipped in mathematics.

It should be stated that all data used are those generally acclaimed in the appropriate fields of study. All through our past up to the Middle ages the time of events is known much better, than population numbers where only orders of magnitude are suggested. But even incomplete data leads to definitive results for interpreting the data of demography, anthropology and history. This finally leads to establishing the concept of the collective quadratic interaction, responsible for the transfer and dissemination of generalized information leading to cooperative human growth and development.

7. Asymptotic solutions and autonomous equations

The asymptotic merging as growth, described as (18a) meets (18c), is best seen if the expansions for the appropriate functions for growth from T_0 and T_1 are compared:

$$n = -\frac{K}{t}\left(1 - \frac{1}{3t^2} + \frac{1}{5t^4} - \cdots\right), \qquad t^2 \geq 1 \qquad (24)$$

$$n = -\frac{K}{t}\left(1 - \frac{t^2}{3K^2} - \frac{t^4}{45K^4} - \cdots\right), \qquad t^2 \le \pi K^2 \tag{25}$$

These functions intersect at a point half way on a logarithmic scale between T_0 and T_1

$$t_{1/2} = -\sqrt{K} \qquad \text{and} \qquad N_{1/2} = K\sqrt{K} \tag{26}$$

at an angle $2/3K$ practically smoothly for any large *K*.

It is obvious that time may be reckoned from T_0, so as to have a solution starting at the beginning of epoch **A** at t_0. Then one can exclude t from (15c), so as to have only one differential equation describing growth and explicitly depending on the state of the system:

$$\frac{dn}{dt} = K \sin^2 \frac{n}{K} + \frac{1}{K} \tag{27}$$

This autonomous equation is valid throughout all times and for large *K* ☜☜1 has a symmetric simplified solution describing the total human story:

$$\tan\frac{n}{K} = \frac{1}{K}\tan\frac{t'}{K} \tag{28}$$

where time t' is reckoned from $t_0 = 0$, with $n_0 = 0$ as the initial condition at t_0.

8. Dynamic stability of growth

In a linear approximation dynamic stability of growth is determined by the growth of fluctuations and any disturbance $\delta n = \delta n_0 \exp \lambda t$ grows or is damped, depending on the sign of the Lyapunov index λ, determined by:

$$\lambda(\nu) = \frac{\partial}{\partial n}\left(\frac{dn}{dt}\right) = \sin 2\frac{n}{K} \tag{31}$$

According to this criteria growth from T_0 to T_1 is unstable as $\lambda \ge 0$ and at $n = (\pi/4)K$, that occurs at $t = -1$, or in 1955 the Lyapunov index reaches its maximum value $\lambda_{max} = 1$. Only after T_1 = 2000, as λ changes sign, does growth stabilize and later stays asymptotically stable.

The Lyapunov index deals with the linear stability of the ‘short’ equations for growth, where all internal processes are averaged. But it is exactly them that can stabilize, or destabilize for that matter, the system. Stabilization can be expected from migration. In 19th century in Europe there were massive migrations to the New World, which certainly stabilized the global demographic system. But the greatest disturbance

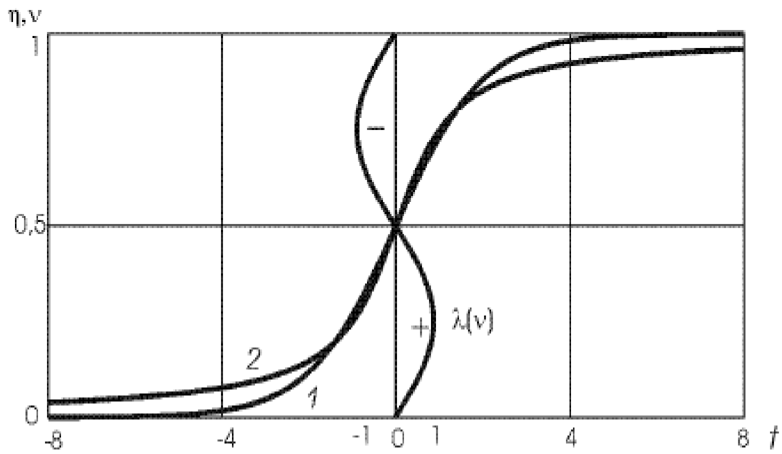

Figure 7. Comparing the logistic – 1 and the demographic transition model – 2 . The logistic approaches zero as an exponent and for $\cot^{-1} t \approx -t^{-1}$ the asymptotic approach in the past is hyperbolic. The Lyapunov index $\lambda(\nu)$, equ.(31) indicates for the model the stability of growth.

during the 20th century – World Wars I and II – are close to the phase of maximum growth and instability of growth to be then expected in the developed countries.

The instabilities were systemic and were due to the internal degrees of freedom in this part of the world, leading to a loss of global stability and 8 – 10% decrease of population. But after the 20th century wars global population came back to its pre-war trend. These considerations indicate that the hyperbolic growth is a path of maximum development – once this track is left, things can only get worse. In complex systems this can be seen as a general principle – as the system is evolving along a deterministic, inherently stable path of maximum growth, most if not all changes, will nearly always make the system less effective. On the other hand, most of the small scale local processes in history contributing to stability are chaotic.

The stability of systems was discussed by Haken (1983) in developing the principles of synergetics and asymptotic methods. There the main motion is identified as *u(t)* – **u**nstable slow motion, in our case *N(T),* and the fast **s**table *s(t)* motion in internal degrees of freedom, that by non-linear coupling stabilizes the main motion. In mechanics this stabilizing effect of internal motion is seen, for example, when a spinning top is stabilized by gyroscopic forces, or when rapid oscillations of the suspension point of a pendulum bring in a stabilizing force.

In the case of population growth a more fundamental understanding can be expected when the internal degrees of freedom will be taken into account in developing the theory of systemic growth. For the stability of the world population system the spatial distribution could then be taken into account. If diffusion is introduced into the kinetic equation one can expect a damping of systemic instabilities, discussed in detail for blow-up in quasilinear parabolic equations by Samarsky et.al (1995).

9. Structure of time, demographic cycles and fluctuations

The change in the scaling of time in human history can be expressed by the instantaneous exponential time of growth T_e. In exponential growth T_e is a constant, but in hyperbolic growth it changes in time:

$$T_e = \left(\frac{1}{N}\frac{dN}{dT} \right)^{-1} = \frac{1}{\tau}\left[(T_1 - T)^2 + \tau^2 \right] \cot^{-1}\frac{T_1 - T}{\tau} \qquad (32)$$

(see equ. 11) or, for the past, before the demographic transition at T_1:

$$T_e(T) \cong T_1 - T \qquad (33)$$

that describes the effective compression of the systemic time of development as T_1 is approached. Exponential time of growth is obviously connected with the Lyapunov index $T_e = 2\tau / \lambda$ for the time of growth of instabilities. After the transition $\lambda \le 0$, the linear stability of growth rapidly rises, reaching its maximum value at $T_1 + \tau$ =2045 and then gradually decreases, but always retaining asymptotic stability, as exponential time of growth rapidly rises $T_e = (T - T_1)^2 / \tau$ and a new temporal structure in the post-transitional world will emerge, probably with a periodic structure based on τ.

Before the transition the periodicity of the demographic cycles appear as a sequence of intervals:

$$\Delta T(\theta) = K\tau \exp(-\theta) \qquad (34)$$

where θ is the number of the period and the integer part of $\ln t' = \ln|t - t_1|$, beginning with $\theta_0 = 0$ and ending at $\theta_1 = \ln K$. Then for the past we obtain:

$$T_1 - T_0 = K\tau \sum_{0}^{\ln K} \exp(-\theta) = K\tau\left[\exp(0) + \exp(-1) + \exp(-2) + \cdots + \exp(-\ln K) \right] \cong$$

$$\cong \frac{e}{e-1} K\tau = 1.582 K\tau \cong \frac{\pi}{2} K\tau = 1.571 K\tau \qquad (35)$$

$\ln K = 11$ cycles of epoch **B** $\Delta P = 2.25 K^2$ = 9 billion people lived as the duration of the cycles shrank from one million to 45 years. The time left after each cycle to T_1 is $(e-1)^{-1} = 0.58$, approximately half the duration of each cycle. The slight difference of the total duration of human development in (21) and (35) is due to growth following a $\tan(t'/K)$ path, and in the second case growth in the past is hyperbolic.

The development at large is stabilized by internal degrees of freedom and the presence of cycles, limited in their amplitude, that indicate the gross stability of the growth of the population system on a scale of T_e. The appropriate periods in Table 2 are well identified and generally recognized by historians and anthropologists. Although no direct demographic data can substantiate such inferences, for even with recent cycles there are no pronounced discontinuities in the growth rates, which are sufficiently significant to mark the onset of a new cycle. Still, these intervals should be called demographic cycles resembling in many ways socioeconomic business cycles.

The results of modeling have to be taken into account in the large scale issues of economic growth and development. Neoclassic economic theory is based on the concept of local equilibrium and of reversible exchanges of goods or services in a market. These processes correspond to the principle of detailed equilibrium of thermodynamics, implying that in a closed system changes are slow and adiabatic. In the case of population growth the rate of changes are determined by the propagation of information vertically – between generations, and horizontally, synchronizing development in space.

As information is irreversibly multiplied by a chain reaction it is not conserved in an open and evolving system. This is the fundamental property of quadratic growth due to a global collective interaction. In other words, the informational and cultural superstructure of society drives the system by an effective interaction moderating all social, cultural, economic and biological factors in a non-equilibrium evolving network of the global population system. The demographic revolution and the onset of exponentially contracting periods in prehistory and history can be seen as a phase transitions of a different scale, punctuating the growth and development of the global population system. After the transition in epoch **C** a more stable and even tranquil stage of global development may by envisaged.

A crude estimate of fluctuations that can be expected in the population of the world is based on the assumption that fluctuations are experienced not by N, but by *n*. In other words, one has to take into account the coherence of structures with a characteristic size of the constant $K \approx 10^5$ – the scale of a unit in population dynamics. Then fluctuations grow as $\delta n \approx \sqrt{n}$. At present they are $\delta N \approx \sqrt{KN} \approx 20 \cdot 10^6$ reaching a maximum $\sim K$ at the end of anthropogenesis and the beginning of epoch **B**.

10. Final remarks

In the model no external restraints appear and this leads to the conclusion that resources, at any rate up to now, do not limit growth, as it is assumed in Malthusian models of the Club of Rome by Meadows (1972) and other global models. If locally resources limit growth this happens not because of a global lack of them, but because of the distribution of wealth, knowledge and resources. What limits growth is the generalized information transfer, the global interaction. This interaction, leading to N^2 growth, is peculiar to humankind, which has acted as an information society right from its very beginning. These new qualities of *Homo sapience* have determined the accelerating pattern of growth of human numbers ever since. Social, cultural and technological development all happened so rapidly, without leaving any time for evolutionary, genetically determined and transmitted changes in human nature, simply because evolution is slow and quadratic growth is ever faster.

In effect, the collective interaction leads to Lamarckian social evolution, when acquired information is directly transmitted between generations mainly by communication and education, and spread worldwide. In biological, Darwinian evolution, information is transmitted genetically and the whole pattern of change is much slower and complex than the social evolution of humankind. Social evolution is happening within a single species now out-numbering all comparable creatures by **a hundred thousand times**. Humans have broken away from nature and, since the Neolithic, have a food base much of our own making. Now growth is culminating in the demographic transition partly because there is no time left to bring up and educate the next generation. This is one of the limits to the multiplication of human numbers contributing to the current informational crisis of growth and development, the TFR crisis of advanced countries. These are the real 'limits to growth' and not energy, space or other resources, as it is commonly assumed. In other words, some 4 to 5 million years ago at the early origins of man began as a predominantly evolutionary systemic process, leading to the appearance of intelligence, language and consciousness. By the end of that period, some half a million years ago, the human characteristics K and τ were attained, at values not that different to what we have now and which have probably not really changed. During all of its growth humanity developed as a global entity, an information society, moderated by processes of obtaining, transferring and multiplying information. Francis Fucuyama stated 'The failure to understand that the roots of economic behavior lie in the realm of consciousness and culture leads to the common mistake of attributing material causes to phenomena that are essentially ideal in nature.'

The demographic revolution is a remarkable transformation, indicating the power of the global interaction, determining growth and development at this most critical stage of the whole story of humankind. The demographic revolution should be seen as the end of self-similar growth, when its 'business as usual' attitude works no longer and informational development has run out of time. Our age is not the age of information, but of the crisis of the information age, the crisis of the values governing our life-style. This is seen in the time and effort that goes into educating the next generation and the informational overload of the modern world. Hence the paradigmatic transition to a new pattern of development, de-coupled from the demographic imperative of numerical growth, where the social 'software' dominates development. This mismatch of our social governance with the brute power of civilization is expressed in the current and, in many ways, decisive crisis of humanity at a very singular time in its history. Thus culture becomes the aim of development, rather than industry and agriculture, which are really only life support systems.

These disparities have contributed to the stress and strain of modern life, to the low number of children in the developed countries of a demographically unsustainable population and the destructive system of values determining our attitudes, discussed by Buchanan (2001). For a stable population the TFR has to be at least 2.15. At present for the developed world TFR ~ 1.4 is well below demographic sustainability. If this trend prevails the native populations of developed countries are heading for extinction. The decrease in TFR is definitely not due to material factors of the 'hardware' of civilization, and can rather be a result of the crisis of the transition itself, to values, the 'software' of our culture. This substantiates the primacy of generalized information as the principle factor in our development

The main global problem facing humanity is to recognize this challenge of the greatest change in human history since the very origin of man, endowed with a brain and mind. The present discussion has brought out the main features of growth of the demographic system that in surprising detail can reveal in quantitative terms the human story, where previously only a descriptive, at best a chronological treatment was possible. The phenomenological theory of growth and development is now open for further studies, as a problem in complexity and evolution of nonlinear systems.

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