'Anticontrol' of Chaos in an Analog Neural Network by External Low-Dimensional Chaotic Force

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Abstract

The results of study of an analog neural network model with time delay which produces chaos similar to the human and animal EEGs are presented. It is shown that the increasing of the neural network asymmetry (increasing of the neurons' excitability) leads to the phenomenon which is similar to the epilepsy. We also consider 'anticontrol' of chaos: the action of the external low-dimensional chaotic force on the neural network with developed epilepsy-like pathological activity. We obtained suppression of pathological nonchaotic activity at the optimal amplitude of external low-dimansional chaotic force. In this case, chaotic neural network demonstrates phenomenon which is similar to stochastic resonance for internal spontaneous oscillations under the action of external low-dimensional chaotic force instead of random noise, as it was studied previously.

1. Introduction

Artificial chaotic neural networks attract particular attention due to thier possible using for control purposes, signal and information processing, memory development, and for understanding of the self-organization processes in the neuron ensembles in the brain.

At present, there are two main hypotheses about the nature of chaos in the brain: low-dimensional chaos and filtered noise. Non-stationary versions of these hypotheses are also discussed.

In early investigations [1–6] it was reported that the human and animal EEGs were produced by low-dimensional chaotic systems. Using Grassberger–Procaccia algorithm, it was shown that the value of the correlation dimension ν was varied from 6–9 in awaking state to 4–5 during sleep or to 2–4 at the epilepsy [2,3,5,6]. The largest Lyapunov exponent value of EEG time series was in the range from 0.028 s^{-1} to 2.9 s^{-1} [1,3,4].

However, more recent investigations [7], using novel method of surrogate data [8], have shown that the evidence for low-dimensional chaotic behaviour is no longer convincing. It was demonstrated that the artificial signal with identical to EEG power spectrum, obtained from the original time series by random mixing of the phases, can mimic low-dimensional chaotic attractors with close values of the correlation dimension [7,8].

Nevertheless, the method of surrogate data was also carefully tested for the Lorentz map and the Hénon map, white Gaussian noises, colored Gaussian noises and mixture of sine waves along with experimental EEG data (alpha-, beta-, delta- and theta-rhythms) in [9]. It is concluded that EEG time series produce finite correlation dimensions. The surrogate testing of eight independent realizations of different forms of EEG activities demonstrates significantly different correlation dimension values than the original data sets (Student's t test) [9].

Establishment of the deterministic origin of the EEG allows to make modelling for the system which produces this signal. Relatively small number of degrees of freedom in EEG points to the high degree of self-organization of the neural electrical activity which changes in dependence on the functional state of the brain (see, for example, [10]). Under such degree of self-organization, it is naturally to suppose that the properties of EEG are essentially determined by the collective degrees of freedom of the neuron outputs. And one can use very simple analog Hopfield neural network model to describe EEG, in which one neuron represents an averaged activity of neural sub-ensemble of the brain.

Early studies of chaotic neural network models are

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mainly devoted to the existence of chaos both in the single neuron output and in the neural ensemble [11,12]. In subsequent investigations, quantitative characteristics of the chaotic behaviour both the single neurons and the neural networks with iterative dynamics (Lyapunov exponent, fractal and information dimensions) were studied [13,14].

Asymmetric analog neural network with time delay and random connectivity was studied in Ref. [15]. It was shown that this neural network produces chaos which is similar to the human or animal EEGs. The correlation dimension ν and the largest Lyapunov exponent λ obtained in Ref. [15] were in the range of experimental values. Changes of the neural network parameters allow to control the degree of chaos and to produce sinusoidal or quasiperiodic outputs. The control of the degree of chaos and transitions "chaos–order", "order–chaos" and "chaos–chaos" with different quantitative characteristics in this neural network were achieved by varying both the amplitude and frequency of the external sinusoidal force too [16]. It was shown in [16] that in the case of resonance, when the frequency of the external force coinsides with one of the eigenfrequencies of spontaneous chaotic activity of the neural network model, considerably smaller amplitudes are necessary to convert chaotic output into sinusoidal one than without resonance. The results of modelling in [16] are in a qualitative agreement with the experiments on controlling chaos in the brain [17–19].

The important role of chaos and noise in the neurons and in the brain is extensively discussed in neurophysiological and physical literature now (see reviews [20,21] and references therein). Main attention is paid to the sensory neurons which allow to encode small environmental changes into spike trains. Recent experiments show constructive role of noise which enhances information transfer by stochastic resonance. Theoretical studies have demonstrated stochastic resonance in different neuron models too.

External noise was also used for control purposes in FitzHugh-Nagumo neuronal model [22] and for suppression of a pathological rhythm in a cardiac model [23].

In this paper, the results of study of an analog neural network model with time delay which produces chaos similar to the human and animal EEGs are presented. Quantitative characteristics of the neural network outputs (correlation dimension, largest Lyapunov exponent, Shannon entropy, normalized Shannon entropy, signal-to-noise ratio (SNR)) are investigated. It is shown that the increasing of the neural network asymmetry (increasing of the neurons' excitability) leads to the phenomenon which is similar to the epilepsy. In this case, in the model, we observe synchronous large-amplitude non-linear oscillations at the all neurons outputs, strong decreasing of the correlation dimension to 1.0–2.2 and zero values of the largest Lyapunov exponents. Obtained values of the correlation dimension in neural network model are close to ones which are observed in the experimental epilepsy studies (2–4). We also consider 'anticontrol' of chaos: the action of the external low-dimensional chaotic force (not the high-dimensional white noise, which have studied previously in [22,23]) on the neural network with developed epilepsy-like pathological activity. Time series for chaotic force are the outputs of this neural network in case of usual (non-pathological) activity with developed low-dimensional chaos and correlation dimensions in the range 5.8–7.5.

2. Model and method of analysis

The asymmetric analog neural network model with the time delay and under the action of external low-dimensional chaotic force which is considered in this paper is described by the set of differential equations:

$$\dot{u}_i(t) = -u_i(t) - \sum_{j=1}^{M} a_{ij} f(u_j(t-\tau_j)) + e_i(t), \qquad i,j = 1,2,...,M, \qquad (1)$$

where $u_i(t)$ is the input signal of the ith neuron, M is the number of neurons, a_{ij} are the coupling coefficients between the neurons, τ_j is the time delay of the jth neuron output, $f(x) = c \cdot \tanh(x-p)$, $e(t)$ is the external low-dimensional chaotic force, p is the threshold of nonlinear function $f(x)$. In this paper the case is studied when the value of τ_j is constant for all neurons ($\tau_j = \tau$). The coupling coefficients are produced by random numerical generator in the interval from -2.048 to $+2.048$, the coefficient c is used in order to vary coefficients a_{ij} simultaneously.

To solve equations (1) the fourth order Runge–Kutta method is used with the time step $h = 0.01$. Small random values of $u_i(0)$ are chosen as the initial values. For the time t in the interval from $-\tau$ to 0 each $u_i(t)$ to be zero. Time series of $N = 100000$ and $N = 8192$ points are analysed since the stationary state is reached after $t_{st} = 10^6 h$. The frequency spectra are calculated using the ordinary digital Fourier transform. For evaluation of the number of the degrees of freedom the correlation dimension ν is calculated.

In order to calculate the largest Lyapunov exponent in M-dimensional phase space two trajectories are computed from equation (1): unperturbed $\mathbf{u}_0(t)$ and perturbed $\mathbf{u}_\varepsilon(t)$ [24]. For the calculation of perturbed trajectory after reaching of the stationary state the small values εu_i are added to u_i. Here ε is in the range from 10^{-14} to 10^{-6}. The largest Lyapunov exponent is defined as

$$\lambda = \lim_{t\to\infty} \lim_{D(0)\to 0} t^{-1} \ln[D(t)/D(0)], \qquad (2)$$

where $D(t)$ and $D(0)$ are the distances between perturbed and unperturbed trajectories at the current and at the initial moments, respectively. The largest Lyapunov exponent λ is calculated from time series of $N = 100000$ points.

To determine the Shannon entropy S_{Sh} characterized the degree of chaos in neural network the interval of the amplitude variation from -4096 to $+4096$ is divided into $K = 128$ subintervals, so that the subinterval length Δ_{Sh} is equal to 64. Then the probabilities p_k to find the trajectory in kth subinterval are calculated. The Shannon entropy is defined by equation

$$S_{Sh} = -\sum_{k=1}^{K} p_k \ln p_k. \qquad (3)$$

Along with S_{Sh}, renormalized Shannon entropy S_r is used. S_r does not depend on the averaged effective energy E, because for the calculation of S_r we renormalize Δ_{Sh} which is used in Eq. (3):

$$\Delta_r = 4\sqrt{E}\Delta_{Sh}, \qquad (4)$$

where

$$E = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{u_i - \overline{u}}{A}\right)^2. \qquad (5)$$

Here $\overline{u}$ is the constant level of u, $A = 4096$ is the normalization constant. Then renormalized Shannon entropy S_r is calculated from Eq. (3), but with the new subinterval length Δ_r defined by Eq. (4). Thus, to determine S_r we use only two momenta of the chaotic signal: the mean value and the dispersion.

The averaged squared amplitude $\overline{u}_{sq}$ in M-dimensional phase space is calculated by equation

$$\overline{u}_{sq} = \frac{1}{AN}\sum_{i=1}^{N}\left(\sum_{j=1}^{M}(u_{ij} - \overline{u}_j)^2\right)^{1/2}. \qquad (6)$$

Signal-to-noise ratio (SNR) is defined as

$$SNR = S_{\omega_0}/S_{noise}, \qquad (7)$$

where S_{ω_0} is the area of Fourier spectrum of the output in the frequency interval $\omega_0 \pm 0.013$ for one main peak (in dimensionless units), S_{noise} is the rest area of Fourier spectrum up to cut-off dimensionless frequency $\omega = 40$. Note, that this SNR definition differs from those which were used in studies of stochastic resonance [20,21], because in our case we have no possibility to determine the level of noise in the spectrum of neuron output. In our neural network model, the spectrum of low-dimensional chaotic signal has many peaks along with very low level of background noise.

3. Simulation results and discussion

Calculations carried out show that asymmetric analog neural network with the time delay and under external action of low-dimensional chaotic force demonstrate periodic, quasiperiodic or chaotic outputs.

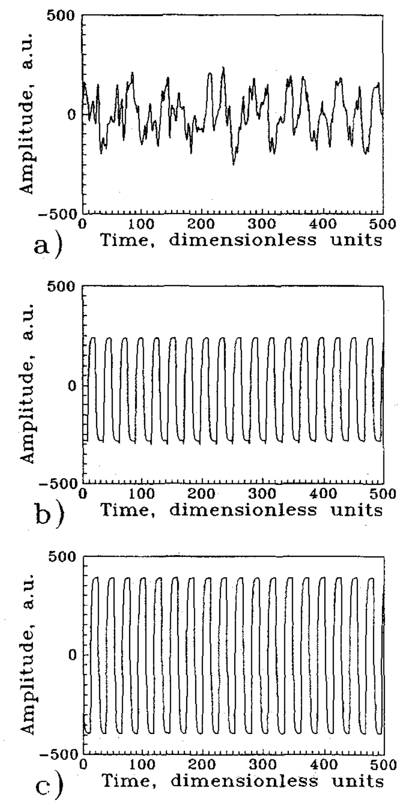

Fig. 1. Time series of the neural network outputs for the 5th neuron, depending on the excitability $\overline{a}$ (M=10, c=3.0, τ=10.0, $e_i(t)$=0.0): $\overline{a} = 0.0$ (a), $\overline{a} = 0.5$ (b), and $\overline{a} = 1.0$ (c).

In the case when amplitude of external forces is zero, the neural network produces chaotic outputs with the correlation dimensions $\nu = 5.8$–7.5 (depending on the ordinal number of the neuron) and the dimensionless largest Lyapunov exponent $\lambda = 0.015$. When we increase the fraction of excitatory coupling coefficients by variation of the average value

$$\overline{a} = \frac{1}{M^2} \sum_{i,j} a_{ij}, \tag{8}$$

from 0 to 0.5, thereby, we replace chaotic neuron outputs by periodic ones. Characteristic time series for the outputs of the 5th neuron in this case are shown in Fig. 1a,b. The largest Lyapunov exponent changes the sign from the positive value to zero one. Further increase of $\overline{a}$ to 1.0 leads only to the increase of neurons' output amplitudes (Fig. 1c). This phenomenon, when the model demonstrate synchronous large-amplitude nonlinear oscillations at the all neurons outputs, strong decreasing of the correlation dimension to 1.0–2.2 and zero values of the largest Lyapunov exponents, is very similar to the epilepsy. Obtained values of the correlation dimension in neural network model are close to ones which are observed in the experimental epilepsy studies (2–4). However, there is disagreement with experimental research of the epilepsy in the signs of the largest Lyapunov exponents: in the experiments, λ has the positive signs [1,3,4].

Figure 2 shows dependence of the neural network output characteristics without an external action (correlation dimension ν, largest Lyapunov exponent λ, Shannon entropy S_{Sh}, renormalized Shannon entropy S_r, and averaged amplitude $\overline{u}_{sq}$) on the degree of neuronal excitability which is characterized by $\overline{a}$. It is seen from Fig. 2 that there are three different intervals of $\overline{a}$ within which the neural network have different types of dynamics. If $\overline{a} = -0.5$ we obtain non-zero constant solution. Increasing $\overline{a}$, the solution becomes unstable, and we have a growth of the correlation dimension ν, largest Lyapunov exponent λ, Shannon entropy S_{Sh} and averaged amplitude $\overline{u}_{sq}$. When $\overline{a}$ reaches value -0.2 and up to $\overline{a} = +0.4$ we obtain the chaotic solutions with the correlation dimension 4–8 which are equal to ones obtained from the human and animal EEG analysis [1–6]. The values of λ, S_{Sh}, S_r and $\overline{u}_{sq}$ do not change esentially on the interval $-0.2 < \overline{a} < +0.4$ too. When $\overline{a}$ exceeds $+0.4$ we observe decreasing of the correlation dimension to 1.0–2.2 and increasing of the averaged amplitude by factor 3–5 (see also Fig. 1) which are the features of the epileptic seizures [1,3,4]. This also leads to the decreasing of λ and S_r that points out to the self-organization processes in neural network model. Note, that decrease of S_r is more clearly shows decrease of neural network output complexity than S_{Sh}.

Similar behavior is observed also in many nonlinear physical systems when growth of energy transfer through the system leads to the replacement of the chaotic transport by the wave one. Such phase transition is the result of the instability development which yields new steady state, in general, with another features.

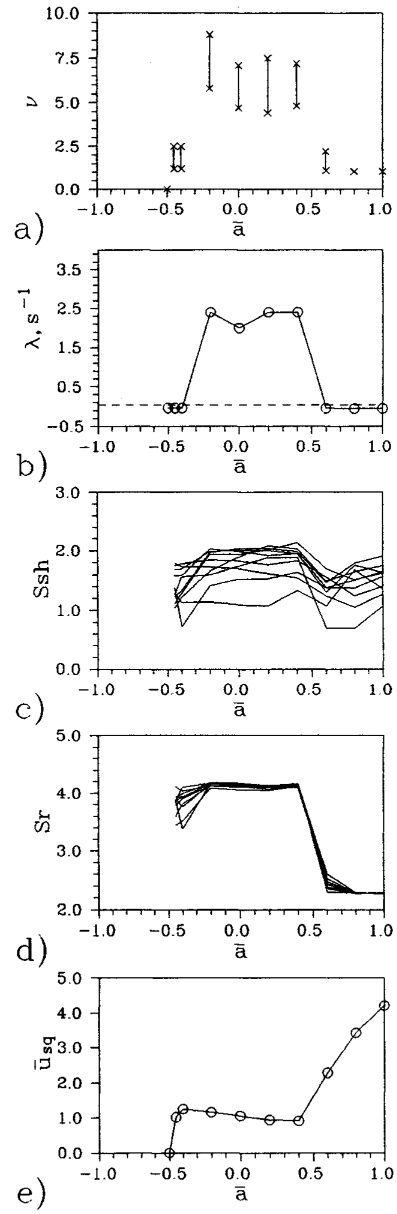

Fig. 2. Correlation dimension ν (a), largest Lyapunov exponent λ (b), Shannon entropy S_{Sh} (c), renormalized Shannon entropy S_r (d), and averaged amplitude $\overline{u}_{sq}$ (e) vs $\overline{a}$: M=10, c=3.0, τ=10.0, $e_i(t)$=0.0.

Figure 3 shows the dependences of the correlation dimension ν, largest Lyapunov exponent λ, Shannon entropy S_{Sh}, renormalized Shannon entropy S_r, and signal-to-noise ratio SNR on the amplitude of the external low-dimensional chaotic force $\mathbf{e}(t)$ in the case of developed pathological rhythmic activity ('anticontrol') when $\overline{a} = 0.5$. As we stated above, the external chaotic force $\mathbf{e}(t)$ is the neuron outputs of the neural network without an external action and at $\overline{a} = 0.0$. The ith component of the external force is applied to the ith neuron in the neural network model. With the increase of the external force amplitude, we observe chaotization of neural network outputs which is indicated by the growth of the correlation dimension, renormalized Shannon entropy and largest Lyapunov exponent. The last quantity also becomes positive. We see that ν, λ and S_r have two maxima in their dependences on e. One maximum occurs at the amplitude of e = 0.6–0.8. In this case we completely restore the degree of chaos in neural network, the correlation dimension ν varying from 4.0 to 7.5 and the largest Lyapunov exponent achieving the values 0.015–0.020.

The next maximum at e = 1.4–1.6 gives only partial restoration of chaos. Here we observe the values of ν and λ which are smaller than those for ordinary spontaneous (non-pathological) neural network activity. Further increase of the amplitude of the external chaotic force produces only non-chaotic neural network outputs.

Thus, in this paper, the existence of the optimal external chaotic force amplitude for the suppression of pathological rhythmic activity have demonstrated for the first time.

If we consider dependence of SNR, as definded in previous section, on e, we observe decrease of SNR for small amplitudes e up to e = 0.8 and then two maxima: one sharp maximum at e = 1.2 almost in all neuron outputs and second broad maximum from e = 1.6 to approximately e = 2.5–3.0 in four neuron outputs. In this case, we observe amplification of SNR for the main frequency peak of intrinsic neural network oscillations by external low-dimensional chaotic force.

4. Conclusions

The results of study of an analog neural network model with time delay which produces chaos similar to the human and animal EEGs show that the increasing of the neural network excitability leads to the phenomenon which is similar to the epilepsy. In this case, in the model, we observe synchronous large-amplitude non-linear oscillations at the all neurons outputs, strong decreasing of the correlation dimension to 1.0–2.2 and zero values of the largest Lyapunov exponents. Obtained values of the correlation dimension in neural network model are close to ones which are observed in the experimental epilepsy studies (2–4).

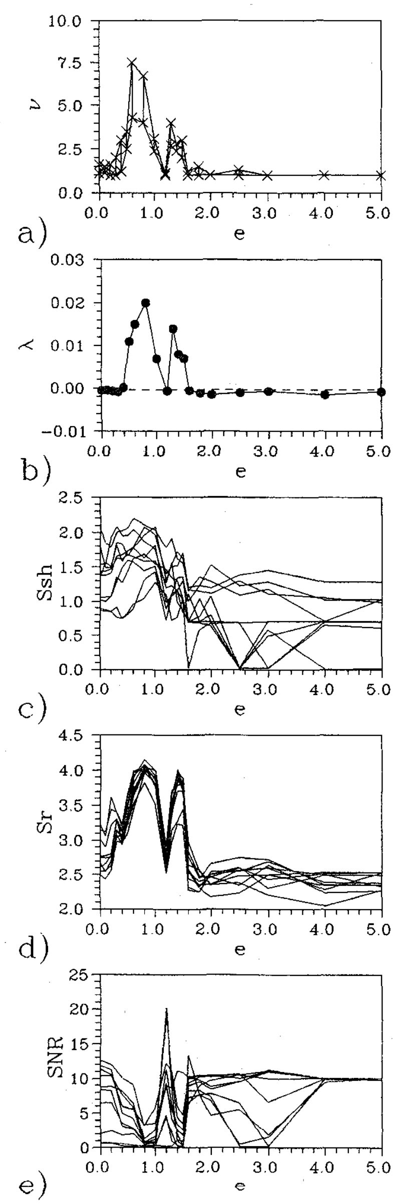

Fig. 3. Correlation dimension ν (a), largest Lyapunov exponent λ (b), Shannon entropy S_{Sh} (c), renormalized Shannon entropy S_r (d), and signal-to-noise ratio SNR (e) vs amplitude of the external low-dimensional chaotic force e: M=10, c=3.0, τ=10.0, $\overline{a}$=0.5.

In the case of 'anticontrol', when we study the action of the external low-dimensional chaotic force on the neural network with developed epilepsy-like pathological activity, we observe 'phase transition' from pathological non-chaotic regime with zero Lyapunov exponent to chaotic one with the positive Lyapunov exponent at the optimal value of amplitude of the external chaotic force. We also obtain the same values of the correlation dimension in the regime of 'anticontrol', as in the regime with usual excitability. 'Anticontrol' of chaos using external chaotic force allows us to suppress immediately pathological activity, rather faster then by drugs.

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