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22 December 1997

Physics Letters A 236 (1997) 513-519

PHYSICS LETTERS A

High-dimensional chaotic neural network under external sinusoidal force

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Received 4 November 1996; revised manuscript received 21 July 1997; accepted for publication 3 October 1997
Communicated by A.R. Bishop

Abstract

Controlling chaos by an external sinusoidal force is considered in an asymmetric analog neural network with time delay. Quantitative characteristics of the chaotic outputs (spectrum, correlation dimension and largest Lyapunov exponent) are studied. It is pointed out that the external sinusoidal force allows one to control the degree of chaos and to produce "chaos-order", "order-chaos", and "chaos-chaos" transitions. Possible mechanisms responsible for the different transitions are discussed. The results of a numerical simulation are compared with the experimental data on controlling chaos in the brain. © 1997 Elsevier Science B.V.

PACS: 87.10.+e; 87.22.−q
Keywords: Chaotic neural network; Correlation dimension; Lyapunov exponent

1. Introduction

Investigations of human and animal EEGs have shown that these signals are deterministic chaotic processes, with the number of degrees of freedom no more than ten, and depend on the functional state of the brain (awake, sleeping, and epilepsy) [1-6]. In order to confirm the deterministic origin of chaos and differentiate it from the quasi-periodicity the largest Lyapunov exponent has been calculated from the experimental time series of the EEG. Its value was in the range from 0.028 s^{-1} to 12.6 s^{-1} [1,3-6].

Establishment of the deterministic origin of the EEG allows one to make a model for the system which produces this signal. The relatively small number of degrees of freedom in an EEG points to the high degree of self-organization of the neural electrical activity, which changes in dependence on the functional state of the brain (see, e.g. Ref. [7]). Under such a degree of self-organization, it is natural to suppose that the properties of an EEG are essentially determined by the collective degrees of freedom of the neuron outputs. One can use the very simple analog Hopfield neural network model to describe the EEG, in which one neuron represents an averaged activity of a neural subensemble of the brain.

Early studies of chaotic neural network models are mainly devoted to the existence of chaos both in the single-neuron output and in the neural ensemble [8-11]. In subsequent investigations, quantitative characteristics of the chaotic behavior of both single neurons and neural networks with iterative dynamics (Lyapunov exponent, fractal and information dimensions) were studied [10,11]. It was shown that the

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PII S0375-9601(97)00796-2

chaotic regimes were observed at the positive values of the largest Lyapunov exponent. Fractal and information dimensions of single-neuron chaotic outputs calculated in Ref. [11] were in the range from 1.5 to 1.8. The asymmetric analog neural network with time delay and random connectivity was studied in Ref. [12]. It was shown that this neural network produces chaos which is similar to human or animal EEGs. The correlation dimension ν and the largest Lyapunov exponent λ obtained in Ref. [12] were in the range of the experimental values. Changes of the neural network parameters allow one to control the degree of chaos and to produce sinusoidal or quasi-periodic outputs. This chaotic neural network also demonstrates epilepsy-like phenomena on increasing the excitability [13].

However, the changes of the neural network parameters in order to control the chaos are not always possible (for example, in the cases of allergy to drug action or under the conditions when rapid changes of the functional state of the brain are necessary). Therefore, it is of interest to study the external action on the neural system too, both in experiments and in the modeling. Moreover, special attention is attracted by investigations of the action of natural and artificial electromagnetic fields on human and animal brains.

Experimental controlling of chaos in the brain was carried out in Ref. [14]. It was shown that the external action on the brain can increase the periodicity of the bursting behavior in the neuronal population. On the other hand, using the "strategy of anticontrol", such systems can be made more chaotic. Quantitative characteristics of the brain response to the external periodic action (correlation dimension and largest Lyapunov exponent) were studied in more detail by Hayashi and Ishizuka in Refs. [5,6]. They found different types of phase-locked and chaotic responses of the brain as caused by external stimulation. The correlation dimension of the field potential oscillations have the values 5.5 ± 0.4 for the spontaneous delta rhythm, 1.5 ± 0.3 for phase-locking and for the non-saturative slope of the correlation integral for the chaotic response between phase-locking, and 2.8 ± 0.2 for chaos which is not a simple mixture of the two kinds of phase-locking. The values of the largest Lyapunov exponent for the different kinds of responses are $3.4 \pm 1.2, 4.4 \pm 1.0, 7.3 \pm 2.1$ and $12.6 \pm 2.9\ \mathrm{s}^{-1}$, respectively [5,6]. Different irregular transitions between these types of responses, in particular between high-dimensional ($\nu > 3$) and low-dimensional ($\nu < 3$) chaos are also observed when the frequency of the external action is varied.

Iterative neural network models under an external action were also studied [15-17]. In Ref. [15], the neural network under the action of external noise was investigated. It was obtained that the external white noise reduces the value of the largest Lyapunov exponent. Controlling the output in chaotic neural networks was studied in Ref. [16]. Changing the parameters of the neural network, it is possible to obtain a transition from chaotic to sinusoidal or constant outputs. In Ref. [17] the threshold parameters were used for control. Deviations of the actual states from the desired states were examined, depending on the range of the coupling coefficients between the neurons. Switching from chaotic to regular outputs has been obtained. However, the study of the changes in the quantitative characteristics of the neural network model outputs in the response to an external periodic force and their comparison with the experimental behavior of the brain has not been performed.

The aim of this Letter is to explain some experimental results on controlling chaos in the brain. A numerical simulation of controlling chaos in an analog neural network with ten neurons under the action of an external sinusoidal force is examined. Quantitative characteristics of the neural network outputs (spectrum, correlation dimension and largest Lyapunov exponent) are investigated in dependence on the amplitude and frequency of the external force. The irregular changes of these characteristics in the response to the external periodic forcing are compared with the ones obtained in the experiments on the periodic stimulation of the brain. Possible mechanisms responsible for different types of neural network outputs are discussed.

2. Model and method of analysis

The asymmetric analog neural network model with time delay and under an external sinusoidal force which is considered in this Letter is described by the set of differential equations

$$\dot{u}_i(t) = -u_i(t) + \sum_{j=1}^{M} a_{ij} f(u_j(t-\tau_j)) + e \sin \omega_e t,$$
$$i, j = 1, 2, \ldots, M, \qquad (1)$$

where $u_i(t)$ is the input signal of the ith neuron, M is the number of neurons, a_{ij} are the coupling coefficients between the neurons, τ_j is the time delay of the jth neuron output, $f(x) = c \tanh(x)$, and e and ω_e are the amplitude and frequency of the external force, respectively. In this Letter the case is studied when the value of τ_j is constant for all neurons ($\tau_j = \tau$). The coupling coefficients are produced by a random numerical generator in the interval from -2.048 to $+2.048$; the coefficient c is used in order to vary the coefficients a_{ij} simultaneously.

To solve Eqs. (1) the fourth order Runge-Kutta method is used with time step $h = 0.01$. Small random values of $u_i(0)$ are chosen as the initial values. For the time t in the interval from $-\tau$ to 0 each $u_i(t)$ is to be taken zero. A time series of $N = 100\,000$ and one of $N = 8192$ points are analyzed since the stationary state is reached after $t_{st} = 10^6$ h. The frequency spectra are calculated using the ordinary digital Fourier transform.

For the evaluation of the number of degrees of freedom the correlation dimension ν_{GP} [18] and the point-wise correlation dimension ν_p [19,20] are calculated, using $N = 8192$ and $N = 100\,000$ points, respectively. In the latter case, $N_{ref} = 100\,000$ points are used, which allows one to achieve the same statistical precision in the calculation of ν_p as at $N_{ref} = N$ [20]. The sampling frequency is chosen so that each significant spectral component should have at least 8-10 points on the time period. Note that the calculated values of the point-wise correlation dimension ν_p from $N = 100\,000$ points do not differ essentially from the correlation dimension ν_{GP} calculated from $N = 8192$ points (deviations are within 15%). This result coincides with Ref. [21], where the calculated values of the correlation dimension from $N = 50\,000$ and $N = 2000$ points, obtained from a chaotic experimental system with $\nu \approx 7$, have the same magnitudes (here and below the letter ν is used to designate ν_{GP} or ν_p without differentiation).

In order to calculate the largest Lyapunov exponent in M-dimensional phase space two trajectories are computed from Eq. (1): the unperturbed $\boldsymbol{u}_0(t)$ and the perturbed $\boldsymbol{u}_\varepsilon(t)$ [22]. For the calculation of the perturbed trajectory after reaching the stationary state the small values εu_i are added to u_1. Here ε is in the range from 10^{-14} to 10^{-6}. The largest Lyapunov exponent is defined as

$$\lambda = \lim_{t\to\infty} \lim_{D(0)\to 0} t^{-1} \ln[D(t)/D(0)], \qquad (2)$$

where

$$D(t) = \left(\sum_{i=1}^{M} [u_{i\varepsilon}(t) - u_{i0}(t)]^2 \right)^{1/2},$$

$$D(0) = \left(\sum_{i=1}^{M} [u_{i\varepsilon}(0) - u_{i0}(0)]^2 \right)^{1/2}$$

are the distances between perturbed and unperturbed trajectories on the current and on the initial moments, respectively. The largest Lyapunov exponent λ is calculated from a time series of $N = 100\,000$ points.

3. Simulation results and discussion

The calculations carried out show that an asymmetric analog neural network with time delay and under the external action of a sinusoidal force demonstrates periodic, quasi-periodic or chaotic outputs.

We start from the case when the amplitude of the external force $e = 0.0$. Under this condition, the neural network produces chaotic output with the correlation dimension $\nu = 5.2$-7.1 (depending on the ordinal number of the neuron) and dimensionless largest Lyapunov exponent $\lambda = 0.017$. Fig. 1 shows the cumulative spectra of ten neuron outputs obtained at $e = 0.0$. We can see that the peak frequencies are in the ratios of 0.12 : 0.28 : 0.46 : 1.04. Similar ratios of the main rhythms of the human EEG (delta-, theta-, alpha-, and beta-rhythms) are also observed in the experiments: 2.3 : 5.5 : 10.5 : 21.5 (see, e.g., Ref. [23]). The correlation dimension ν calculated from Eq. (1) is also in the range of the experimental values $\nu = 6$-9. To compare the calculated value of λ with that obtained from a human EEG we perform a time normalization, so that the third frequency peak of the numerical solution (Fig. 1) is taken to be equal to the frequency of the human α-rhythm, $f_\alpha = 10.5$ Hz. In this case, the largest Lyapunov exponent obtained from the numerical solution is equal to 2.4 s^{-1} and agrees with the experimental values up to 12.6 s^{-1} [1,3-6].

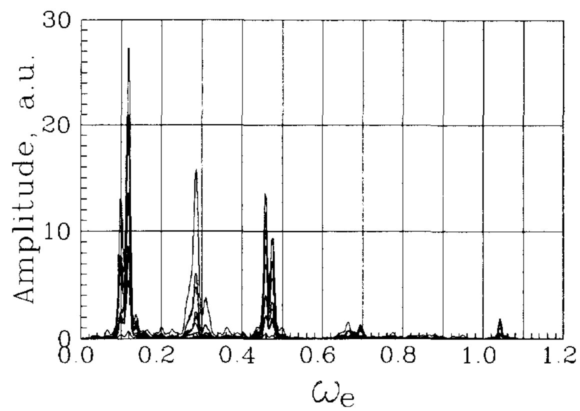

Fig. 1. Spectra of the outputs for all ten neurons without an external action; $M = 10, c = 30, \tau = 10.0, e = 0.0$.

Thus, we see that the chaotic neural network under study has quantitative non-linear characteristics which are similar to the human EEG.

When we apply to this neural network an external sinusoidal force, we can change its output from relatively high-dimensional chaotic ($\nu \sim 5$–$8, \lambda > 0$) to low-dimensional chaotic ($\nu \leqslant 3, \lambda > 0$), quasi-periodic ($\nu \leqslant 3, \lambda \approx 0$) or periodic ones ($\nu \approx 1$, $\lambda \approx 0$). The changes of these regimes are irregular with the growth of the external force frequency and qualitatively reflect the experimental observations in Refs. [5,6].

Plots of the maps, time series and Fourier spectra for these different regimes for the sixth neuron are shown in Fig. 2. We clearly see a qualitative difference of the regimes. The output of the sixth neuron without an external action has the correlation dimension $\nu = 5.4$. This value of the correlation dimension coincides with $\nu = 5.5 \pm 0.4$ for the spontaneous field potential oscillations of Ref. [6].

When we apply the external force with a frequency $\omega_e = 0.1$ and amplitude $e = 7.0$, the output of the neural network shows low-dimensional chaos with $\lambda = 0.004$ and $\nu \approx 2.5$ for the sixth neuron. This case corresponds to the strong resonant action of the external force on the neural network when the frequency of the external force is very close to the eigenfrequency of spontaneous oscillations with a relatively large amplitude (first peak in Fig. 1). This action also produces the largest output amplitude; see Figs. 2a,e). The spectrum of the output has the largest peak at the frequency $\omega_e = 0.1$ and three smaller peaks of its second, third, and fourth harmonics, which frequencies are slightly smaller than 0.2, 0.3, and 0.4, respectively (Fig. 2i). This type of spectrum is characteristic for the quadratic and fourth power mode coupling, with subsequent destruction of quasi-periodicity and chaotization of output (see Ref. [24]). In this case, spontaneous oscillations are considerably suppressed by the main oscillation with $\omega_e = 0.1$. As a result, we have an attractor with a correlation dimension which is smaller than the value of ν for spontaneous oscillations. This regime corresponds to the case of chaos, with the correlation dimension $\nu = 2.8 \pm 0.2$ in Ref. [6]. Note that a similar spectrum with one largest low-frequency peak, small correlation dimension and positive largest Lyapunov exponent is also observed in Ref. [1] in a study of epilepsy.

The second case, Figs. 2b,f,j, corresponds to the quasi-periodic solution. It has $\lambda \approx 0$ within the accuracy of calculations $|\Delta\lambda| \leqslant 0.001$ and $\nu \approx 2.2$. In this case we also observe resonance, but it is less strong, because the difference between the frequency of the external force and the first peak of the resonant frequency for spontaneous oscillations is greater than in the previous case. Here the cubic non-linearity plays an important role, as is seen from the spectrum in Fig. 2j, when the first and third harmonics have the greatest amplitudes along with weak combinatory frequencies' peaks. However, in this case, the largest peak amplitude is considerably smaller than in the previous regime. It suppresses chaos, but does not allow one to destroy quasi-periodicity. In spite of that this regime is also low-dimensional; it has a more ordered structure, as seen from comparison of the maps in Figs. 2a and 2b. It is close to the regime of phase-locking in Refs. [5,6], if we compare corresponding maps. The positive largest Lyapunov exponent obtained in the regime of phase-locking, as is emphasized in Ref. [6], appears due to the influence of noise.

In the third case, the external force with $\omega_e = 0.6$ (Figs. 2c,g,k), produces the most chaotic map in Fig. 2. Here, the largest Lyapunov exponent and correlation dimension have relatively large values, $\lambda = 0.023$ and $\nu \approx 7.7$, respectively. Owing to the absence of resonance, the external action with this frequency only slightly modifies the spontaneous spectrum, introducing oscillations with the dimensionless frequencies 0.6 and 1.2 (Fig. 2k). However, by increasing the amplitude of the external force, we

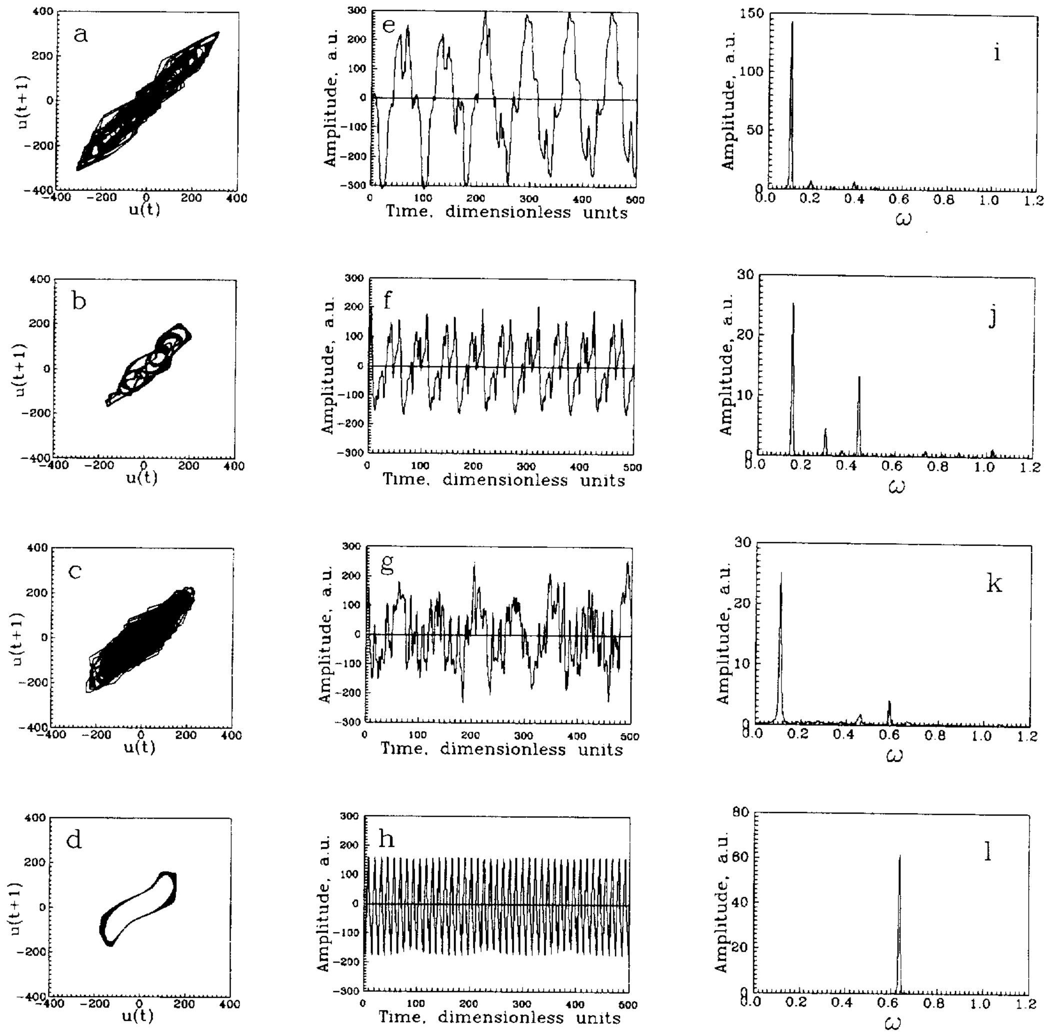

Fig. 2. Maps (a)–(d), time series (e)–(h) and spectra (i)–(l) of the different regimes of the neural network output for the sixth neuron ($M = 10, c = 3.0, \tau = 10.0, e = 7.0$): low-dimensional chaos, $\omega_e = 0.1$ (a), (e), (i); quasi-periodic output, $\omega_e = 0.15$ (b), (f), (j); high-dimensional chaos, $\omega_e = 0.6$ (c), (g), (k), and periodic output, $\omega_e = 0.65$ (d), (h), (l).

can suppress chaos in this case too (see a similar case below in Figs. 4c,d). This most chaotic regime corresponds to the case between phase-locking in Ref. [6], when two different regimes co-exist. In Ref. [6] saturation of the slope of the correlation integral in this regime has not been obtained.

Finally, the last case with periodic output, Figs. 2d,h,l, corresponds to the case of phase-locking in Ref. [6]. Our calculations give $\lambda \approx 0$ and $\nu = 1.05$. Here we have resonant interaction of the external force ($\omega_e = 0.65$) with the small peak at the same frequency in the spontaneous spectrum (Fig. 1). This interaction suppresses chaos and produces slightly modulated sinusoidal oscillations, which are clearly seen in Figs. 2d.h,l.

The correlation dimension ν and the largest Lyapunov exponent λ as functions of the sinusoidal external force frequency ω_e are shown in Fig. 3. It is seen that both ν and λ are irregular functions of the frequency ω_e. In Fig. 3a we observe the transitions between relatively low-dimensional ($\nu \leqslant 3$) and high-dimensional ($\nu \sim 5$–8) chaotic regimes, the largest

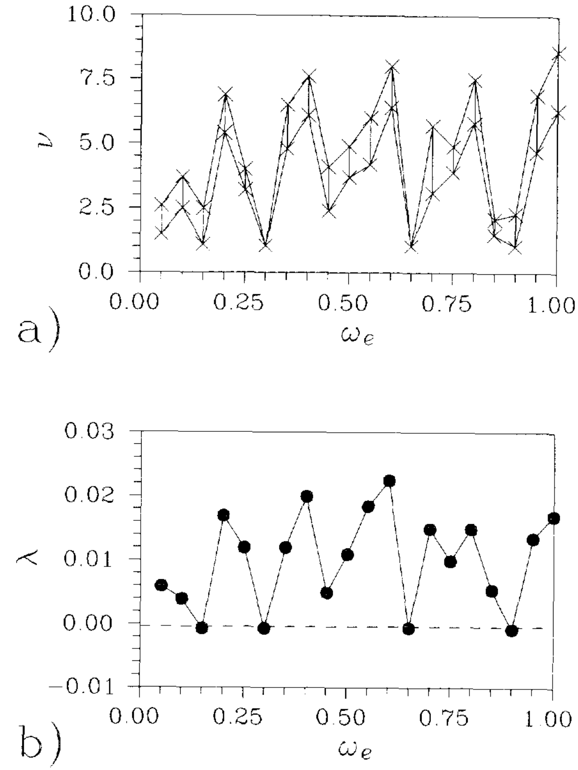

Fig. 3. Correlation dimension λ (a) and largest Lyapunov exponent λ (b) as functions of the external force frequency ω_e, $M = 10, c = 3.0, \tau = 10.0, e = 7.0$. For the correlation dimension the intervals in which ν is varied are shown.

Lyapunov exponent having larger values in the high-dimensional cases than in the low-dimensional ones (Fig. 3b). As a rule, low-dimensional outputs are observed when the frequency of the external force is close to the eigenfrequency of self-excited oscillations in the neural network without an external action. We obtain quasi-periodic or periodic outputs with $\lambda \approx 0$, when ω_e is close to the eigenfrequency of self-excited oscillations too. Fig. 3 is in a qualitative agreement with Fig. 8 from Ref. [6], where chaotic regimes alternate with phase-locked ones.

Thus, varying the frequency of the external sinusoidal force, it is possible to control the number of degrees of freedom and the largest Lyapunov exponent (2) in the chaotic neural network outputs. Thereby we can also produce "chaos–order", "order–chaos", and "chaos–chaos" transitions with different quantitative characteristics in the neural network model, which are observed in the experimental control of chaos in the

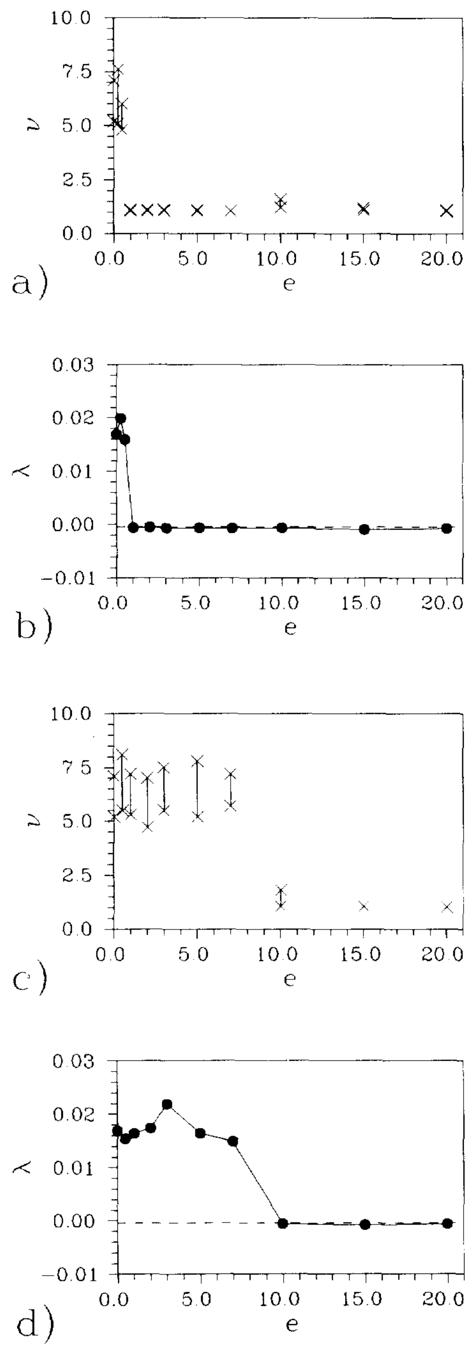

Fig. 4. (a), (c) Correlation dimension ν and (b), (d) largest Lyapunov exponent λ as functions of the external force amplitude e; $M = 10, c = 30, \tau = 10.0$; (a), (b) $\omega_e = 0.3$, (c), (d) $\omega_e = 0.8$.

brain [5,6].

Fig. 4 shows the correlation dimension ν and the largest Lyapunov exponent λ versus amplitude of the external force e at two fixed values of ω_e: 0.3 and 0.8. The first value of ω_e corresponds to the resonance in the neural network (as seen in Fig. 1), the second value does not produce a resonant action. Both cases show suppression of chaos in the neural network with the increase of the external force amplitude. However, in the case of resonance, considerably smaller amplitudes are necessary to convert a chaotic output into a sinusoidal one than without resonance. This is clearly seen in the behavior of the largest Lyapunov exponents in Figs. 4b,d. The dependence of the correlation dimensions ν on the amplitude e demonstrates controlling of chaos too, decreasing from approximately 5–8 to 1 with the growth of e (Figs. 4a,c). The experiments (Fig. 8) in Ref. [6] also show that in the case of resonance we should apply a smaller external stimulation to suppress a spontaneous chaotic activity. If we increase the frequency of the stimulation, we must also increase the stimulus intensity to suppress chaos and to obtain non-chaotic regimes of phase-locking.

4. Conclusions

Thus, the results of this work have demonstrated controlling the degree of chaos by an external sinusoidal force in neural networks with relatively high correlation dimensions, comparable to that of the human EEG. The control of the degree of chaos and "chaos–order","order–chaos" and "chaos–chaos" transitions with different quantitative characteristics are produced by varying both the amplitude and the frequency of the external force. The results of this modeling allow one to explain the experimental results on controlling chaos in the brain.

Acknowledgement

The author would like to thank O.S. Bartunov and S.I. Blinnikov from Supernovae Research Group (Sternberg State Astronomical Institute) for support of this research.

References

[1] A. Babloyantz, A. Destexhe, Proc. Nat. Acad. Sci. USA 83 (1986) 3513.
[2] A. Destexhe, J.A. Sepulchre, A. Babloyantz, Phys. Lett. A 132 (1988) 101.
[3] A.C.K. Soong, C.I.J.M. Stuart, Biol. Cybern. 62 (1989) 55.
[4] G.W. Frank, T. Lookman, M.A.H. Nerenberg, C. Essex, J. Lemieux, W. Blume W, Physica D 46 (1990) 427.
[5] H. Hayashi, S. Ishizuka, Brain Res. 686 (1995) 194.
[6] S. Ishizuka, H. Hayashi, Brain Res. 723 (1996) 46.
[7] W.J. Freeman, Sci. Am. 264 (1991) 78.
[8] K.E. Kürten, J.W. Clark, Phys. Lett. A 114 (1986) 413.
[9] W.J. Freeman, Biol. Cybern. 56 (1987) 139.
[10] K. Aihara, T. Takabe, M. Toyoda, Phys. Lett. A 144 (1990) 333.
[11] M. Nakagawa, M. Okabe, J. Phys. Soc. Jpn. 61 (1992) 1121.
[12] V.E. Bondarenko, Phys. Lett. A 196 (1994) 195.
[13] V.E. Bondarenko, in: The 1996 IEEE Int. Conf. on Neural Networks, Vol. 2, Washington, DC, USA (New York, IEEE, 1996) p. 774.
[14] S.J. Schiff, K. Jerger, D.H. Duong, T. Chang, M.L. Spano, W.L. Ditto, Nature 370 (1994) 615.
[15] L. Molgedey, J. Schuchhardt, H.G. Schuster, Phys. Rev. Lett 69 (1992) 3717.
[16] K. Nakamura, M. Nakagawa, J. Phys. Soc. Jpn. 62 (1993) 2942.
[17] J. Warncke, M. Bauer, W. Martienssen, Europhys. Lett. 25 (1994) 323.
[18] P. Grassberger, I. Procaccia, Phys. Rev. Lett. 50 (1983) 346.
[19] J. Theiler, J. Opt. Soc. Am. A 7 (1990) 1055.
[20] J. Theiler, Phys. Rev. A 41 (1990) 3038.
[21] M. Ding, C. Grebogi, E. Ott, T. Sauer, A. Yorke, Phys. Rev. Lett. 70 (1993) 3872.
[22] H.G. Schuster, Deterministic Chaos (Physik-Verlag, Weinheim, 1984).
[23] Principles of Neural Science, eds. E.R. Kandel, J.H. Schwartz, T.M. Jessel, 3rd Ed. (Appleton & Lange, Norwalk, CT, 1991) p. 778.
[24] V.S. Anishchenko, Complex Oscillations in Simple Systems (Nauka, Moscow, 1990), Chapter 12.